Timing dissipative quantum processes

An invitation

Michael J. Kastoryano

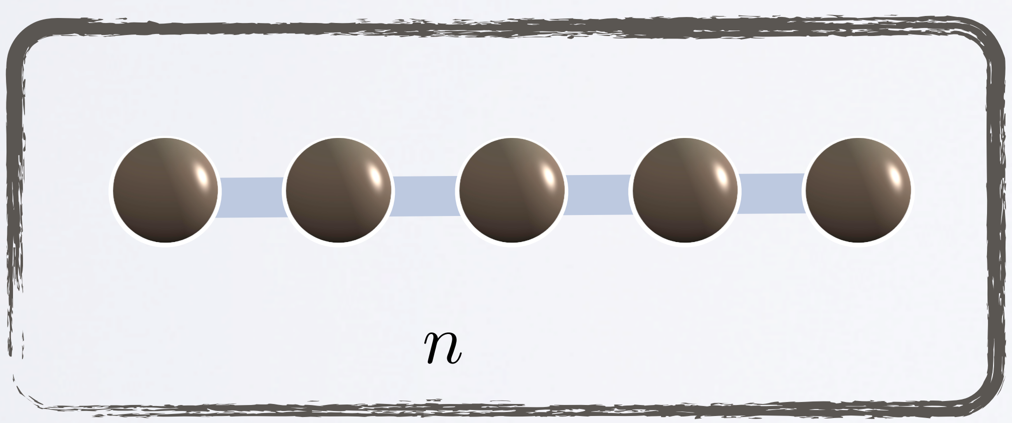
Freie Universität Berlin
Dahlem Center for Complex Quantum Systems

Joint work with Jens Eisert and Michael M Wolf, PRL 110, 110501 (2013)

DPG, March 2013

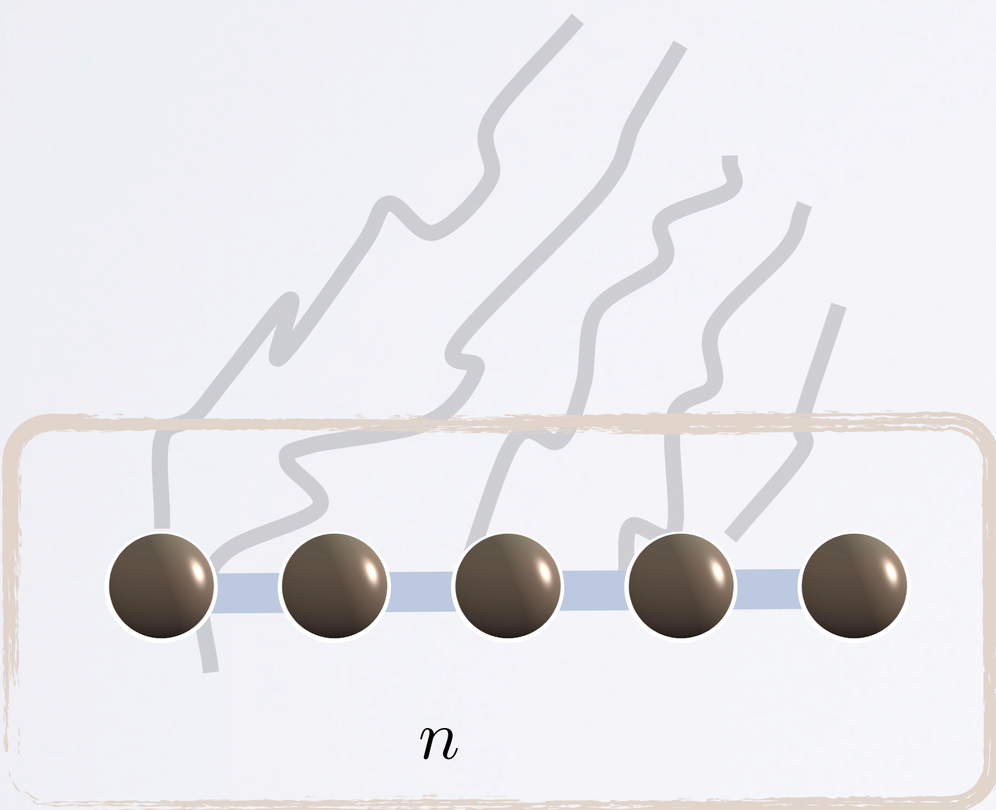
Motivation

- Shielding systems in QI from their environment



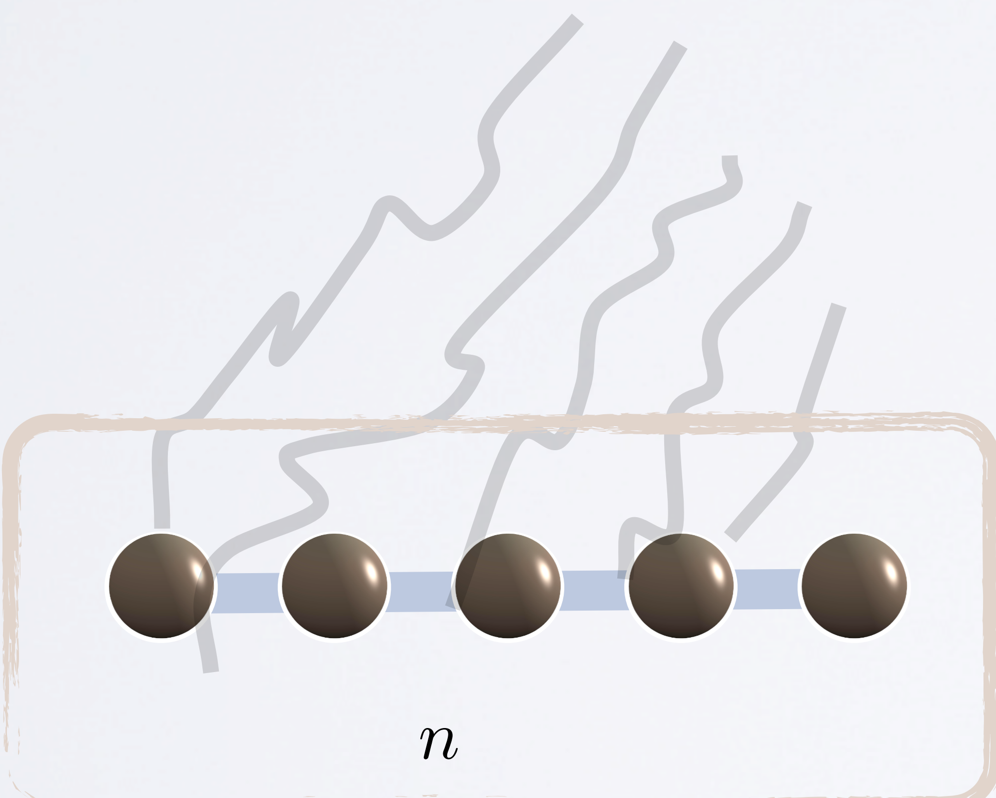
Motivation

- Open systems dynamics



Motivation

- Weak quantum noise is often to good approximation **Markovian**



- **Dynamical map** $T_t = e^{t\mathcal{L}} : \mathcal{M}_d \to \mathcal{M}_d$
  gives rise to semi-group, L-GKS- theorems give

$$\mathcal{L}(\rho) = i[\rho, H] + \sum_j \left(L_j^\dagger L_j \rho + \rho L_j^\dagger L_j - 2 L_j \rho L_j^\dagger \right)$$

- Natural: k -**local** quantum noise:

$$\mathcal{L} = \sum_j \mathcal{L}_j$$

- Lindblad, Commun Math Phys **48**, 119 (1976)
- Gorini, Kossakowski, Sudarshan, J Math Phys **17**, 821 (1976)
- Cubitt, Eisert, Wolf, Commun Math Phys **310**, 383 (2012)
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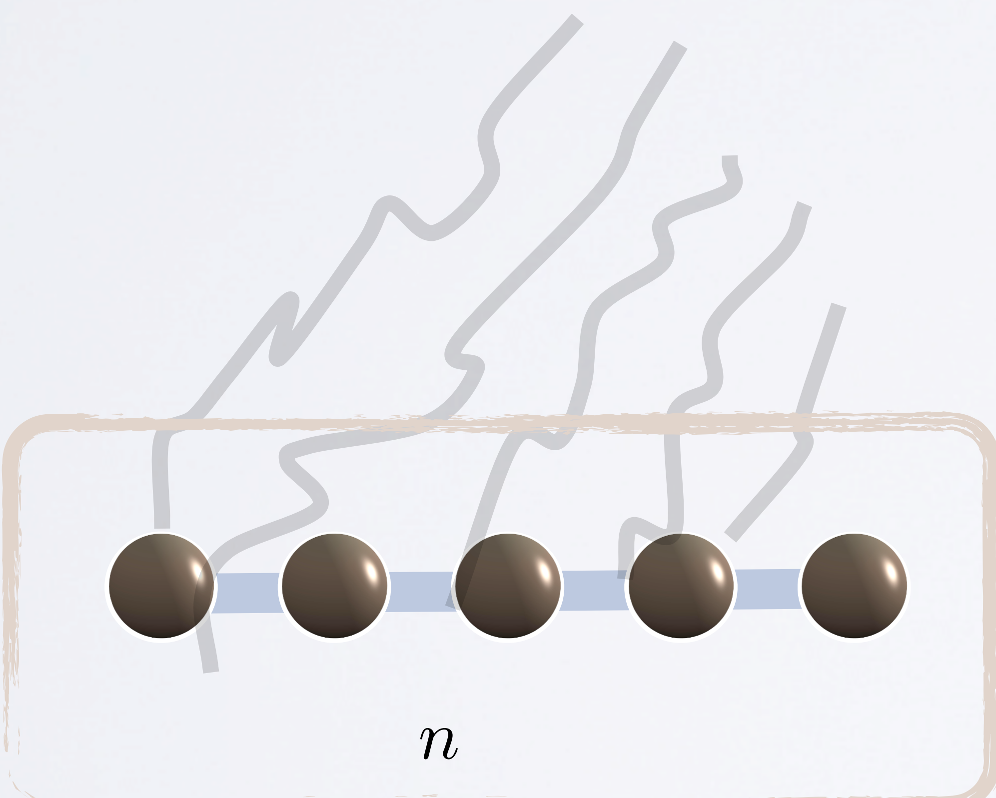

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- Natural: k -**local** quantum noise:

$$\mathcal{L} = \sum_j \mathcal{L}_j$$

- Matrix form

$$\hat{\mathcal{L}} = \sum_j \left(2 L_j \otimes L_j^* - L_j^\dagger L_j \otimes 1 + 1 \otimes L_j^\dagger L_j \right)$$

- **Gap** λ : smallest non-zero real part of eigenvalues

- Lindblad, Commun Math Phys **48**, 119 (1976)
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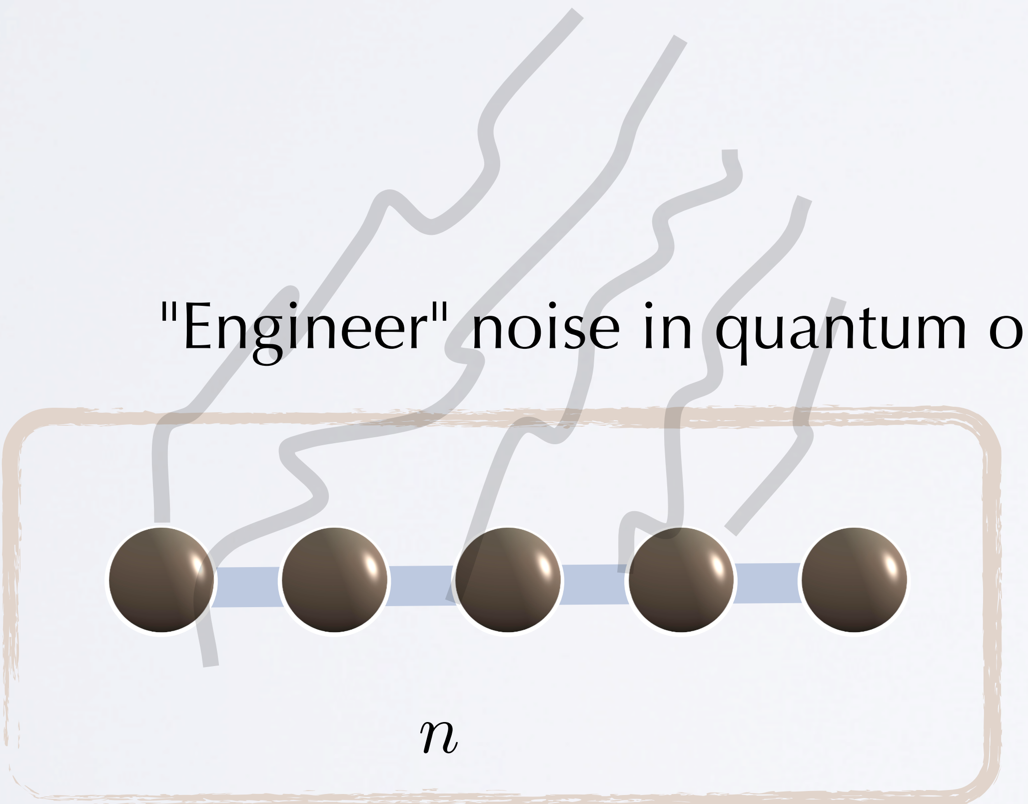
Dissipation-driven quantum information processing

- **Make noise an ally: Dissipation-driven quantum information processing**

- Steady-state ρ_{ss} can be pure (dark state), unique, reachable in poly time

"Engineer" noise in quantum optical systems: To extraordinary extent Markovian



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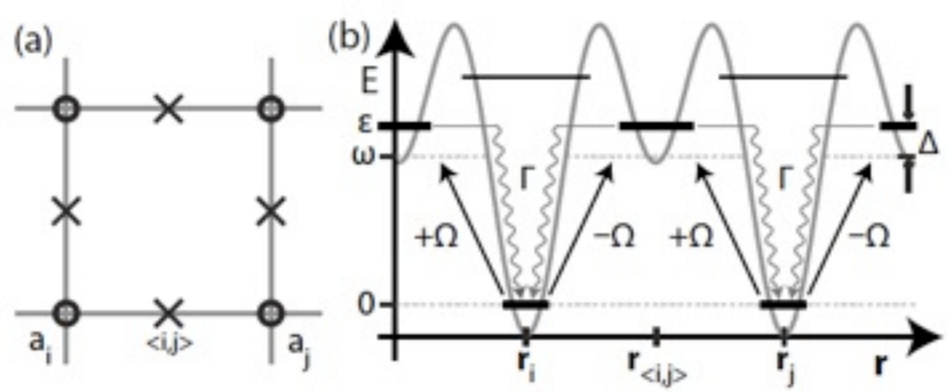
- Dissipative state preparation, driven criticality, topology by dissipation



- Diehl et al, Nat Phys **4**, 878 (2008)
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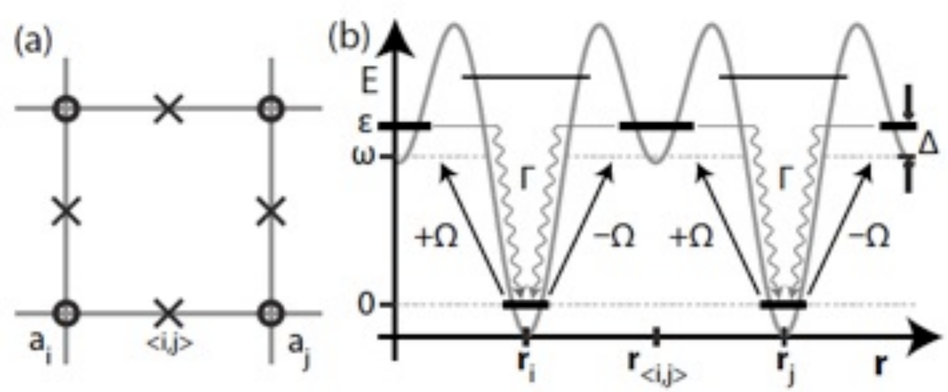

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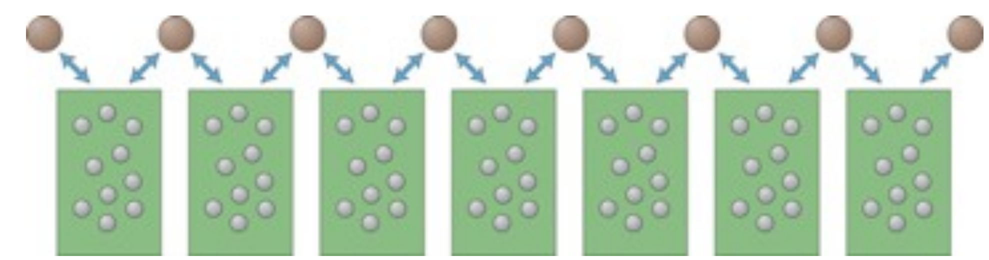
- Dissipative quantum computing



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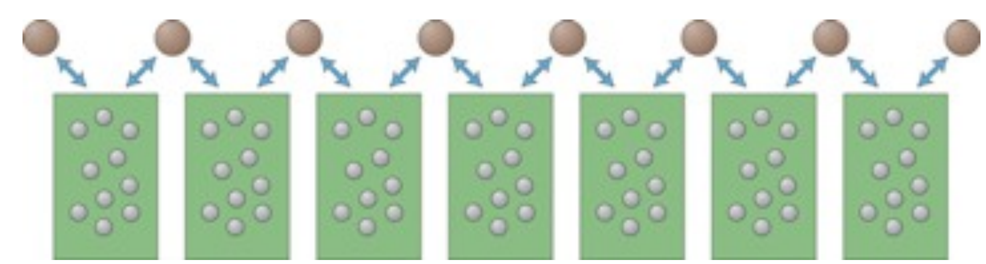

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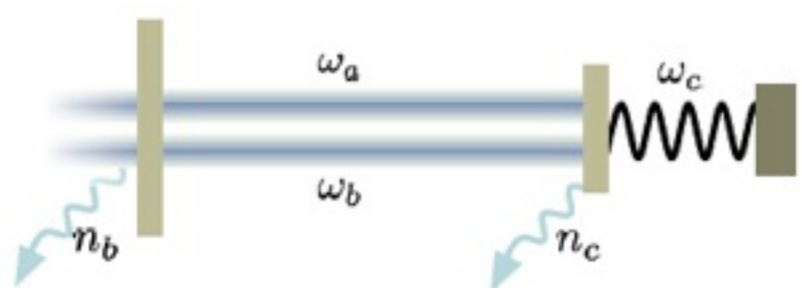
- Cooling by heating and dissipation



- Mari, Eisert, Phys Rev Lett **108** (2012)

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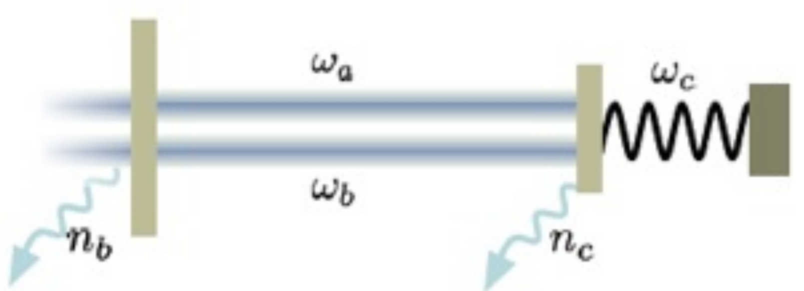

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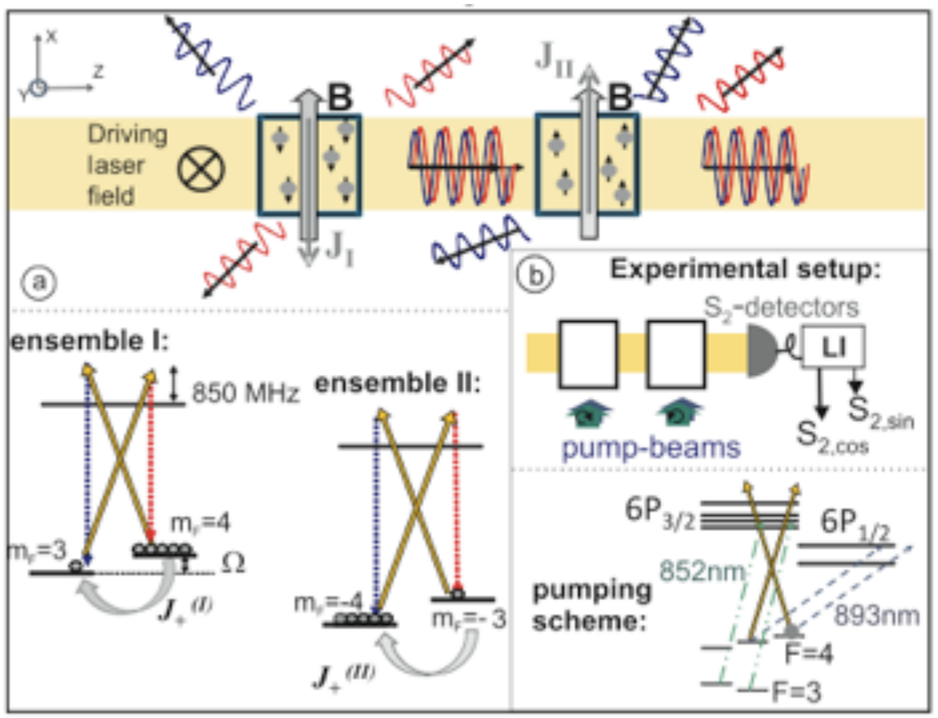
- Dissipative quantum computing

- Experiments with entanglement-generation by dissipation



- Barreiro et al, Nature **470**, 486 (2011)
- Krauter et al, Phys Rev Lett **107**, 080503 (2011)

But...

- **Make noise an ally: Dissipation-driven quantum information processing**

- Cannot so easily measure, interact, do things sequentially, condition on outcomes, do quantum error correction, think of quantum memories?
- Easier proof techniques?

The cutoff phenomenon

The cutoff phenomenon

The cutoff phenomenon

- How long does the process run before reaching equilibrium?

- **Convergence theorem:**
Let $T_t : \mathcal{M}_d \to \mathcal{M}_d$ be a Markovian quantum channel whose Liouvillian has a gap λ. Then for any $\nu < \lambda$ and for any $\rho_0 \in \mathcal{S}_d$ there exists a real constant R such that for all $t \geq 0$

$$\|T_t(\rho_0) - \rho_{ss}\|_1 \leq Re^{-t\nu}$$

- For rapid convergence, we need for $\log d \sim n$

$$\nu^{-1} < \text{poly}(n)$$

$$R < e^{\text{poly}(n)}$$

- Pre-asymptotic behavior can be highly non-trivial!

The cutoff phenomenon

- Cutoff-phenomenon for **classical Markov chains**
- Deck of n cards: • Well mixed after $3/2 \log n$ shuffles

(*Gilbert Shannon model*)

- Diaconis, Proc Natl Acad Sci USA **93**, 1659 (1996)
- Williams, Magician Monthly **8**, 67 (1912)

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  - Very poorly mixed before $3/2 \log n$ shuffles

(*Gilbert Shannon model*)

- At basis of magic trick *"Premo"*

$\|P_x^k - p_{ss}\|$

k									
1	2	3	4	5	6	7	8	9	10
1.000	1.000	1.000	1.000	0.924	0.624	0.312	0.161	0.083	0.041

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1.000	1.000	1.000	1.000	0.924	0.624	0.312	0.161	0.083	0.041

- Some information about the initial state is preserved until a **critical number of shuffles (7)**
- Shortly afterwards, essentially **no** information can be recovered

- Diaconis, Proc Natl Acad Sci USA **93**, 1659 (1996)
- Williams, Magician Monthly **8**, 67 (1912)

The cutoff phenomenon

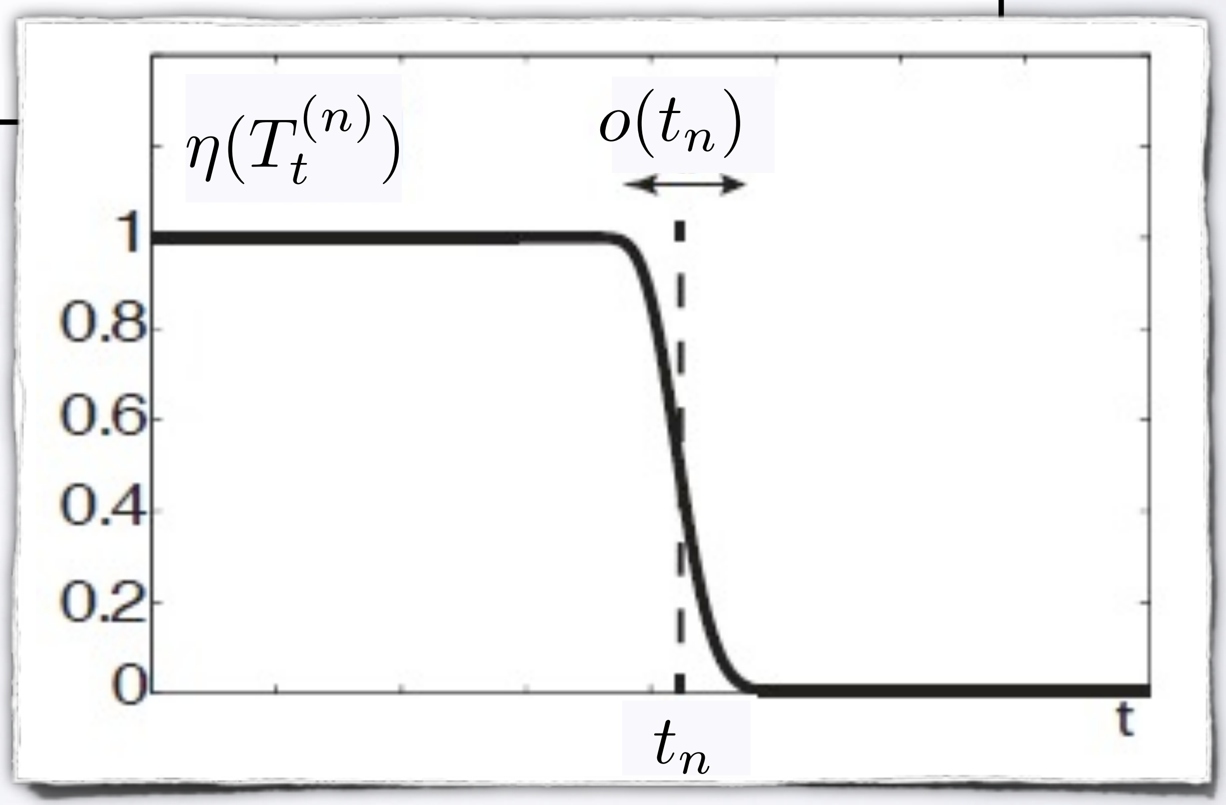
- Convergence measure: $\eta(T) = \frac{1}{2} \sup_{\rho \in \mathcal{S}_d} \|T(\rho) - \rho_{ss}\|_1$ (unique ss)

- **Cutoff phenomenon:**

Let $T_t^{(n)}$ be a sequence, indexed by the system size n, of Markovian quantum channels. We say that $T_t^{(n)}$ exhibits a cutoff at times t_n if for any $c > 0$

$$c < 1 \Rightarrow \lim_{n\to\infty} \eta(T_{ct_n}^{(n)}) = 1$$

$$c > 1 \Rightarrow \lim_{n\to\infty} \eta(T_{ct_n}^{(n)}) = 0$$



- Kastoryano, Reeb, Wolf, arXiv:1111.2123
- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

The cutoff phenomenon

- **Idea:** Use in quantum version, construct timing gadgets from it!

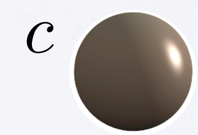
- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Initialization gadget

Initialization gadget

Initialization gadget

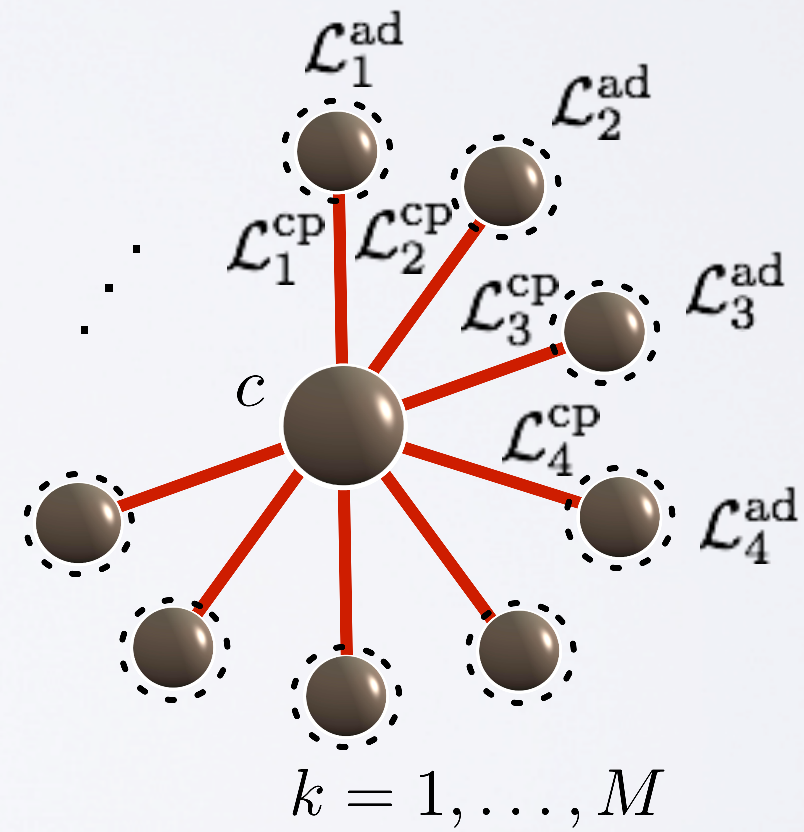
- Central qubit c should be prepared in $|0\rangle_c$
- Amplitude damping channel does the job, but will not stop preparing!



- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Initialization gadget

- Central qubit c should be prepared in $|0\rangle_c$ in star graph
- Apply Liouvillian with Lindblads $L_k^{\rm ad} = \sqrt{\omega}|0\rangle_k\langle 1|_k$ to outer qubits
- Couple with Liouvillian with Lindblads $L_k^{\rm cp} = \sqrt{\gamma}|0\rangle_k\langle 1|_k \otimes |1\rangle_k\langle 1|_k$



- Intuition: Stops preparing after time $t_M \approx \log M/\omega$

- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

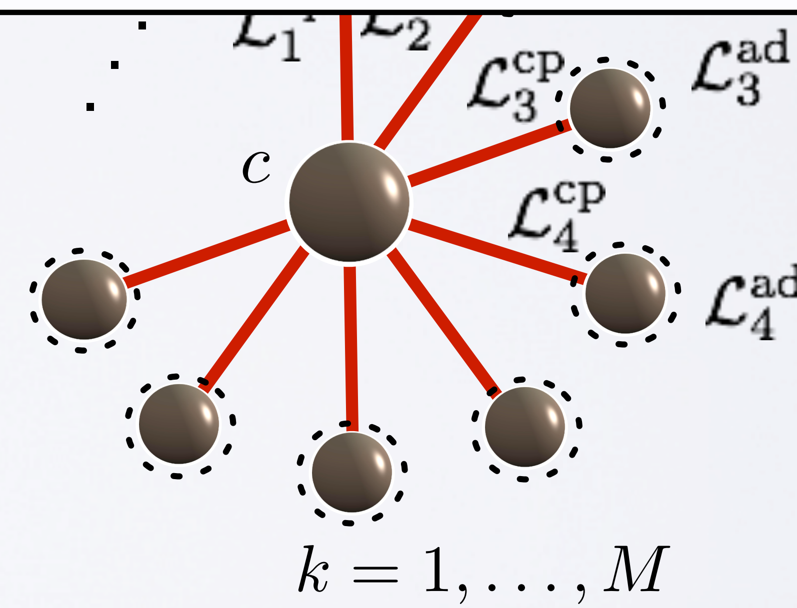
Initialization gadget

- **Lemma (Initialization):**

  Let ρ be an arbitrary input state. If there exist $\delta, c > 0$ and a subset $S \subset \{1, \ldots, M\}$ with $|S| = cM$ such that $\langle 1_j|\rho_j|1_j\rangle > \delta$ for all $j \in S$. Then for any $\varepsilon > 0$ there ex a $\tau = O(\log(M))$ such that for all $t > \tau$

$$\langle 1_c|\rho_c(t)|1_c\rangle \leq Me^{-\mu M} + \varepsilon$$

  where ρ_j is the reduced state of subsystem j and $\rho_c(t) = \mathrm{tr}_{\mathrm{aux}} e^{t\mathcal{L}^{\mathrm{ini}}}(\rho)$ is the partial trace over the auxiliary qubits, and μ is some constant depending on $\{c, \delta, \omega, \Gamma\}$



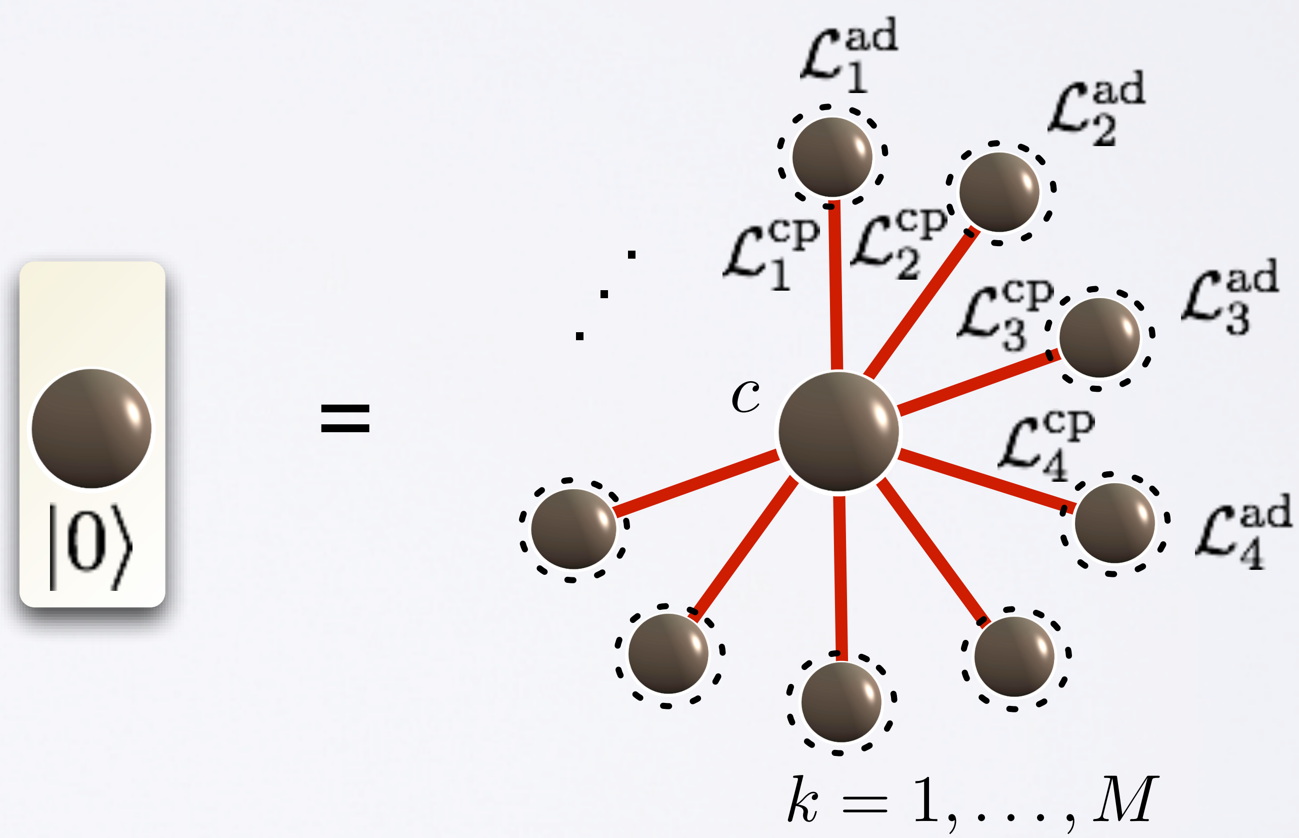
- *Can prove:* All (except exp small subset) initial states yield state *exponentially close* to

$$\rho_0 := |0\rangle_c\langle 0|_c \otimes |0\rangle\langle 0|^{\otimes M}$$

- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Initialization gadget

- Rigorous error bounds, for all initial states **exponentially small** in M
- Gives **initialization gadget**



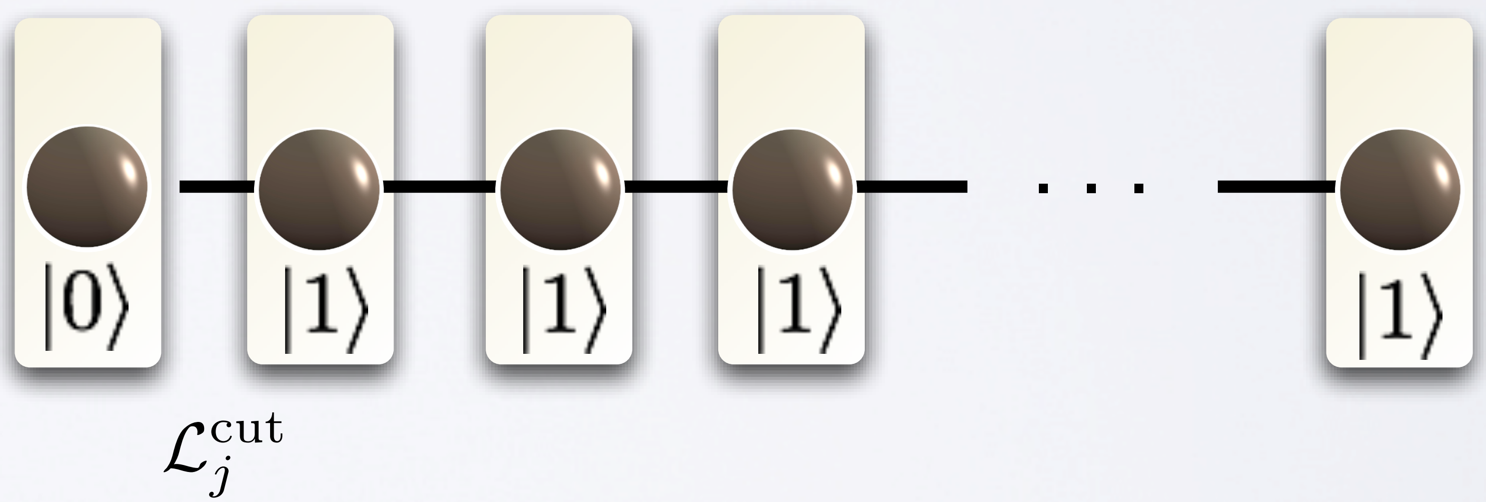
- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Timer gadget

Timer gadget

Timer gadget

- Take now linear chain of N qubits
- Initialize (as above)
- Couple with local Liouvillian with Lindblads $L_j^{\text{cut}} = \sqrt{\gamma}|0\rangle_j\langle 0|_j \otimes |0\rangle_{j+1}\langle 1|_{j+1}$



- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Timer gadget

- **Lemma (Timing)**

Let N be the number of timer qubits, then for

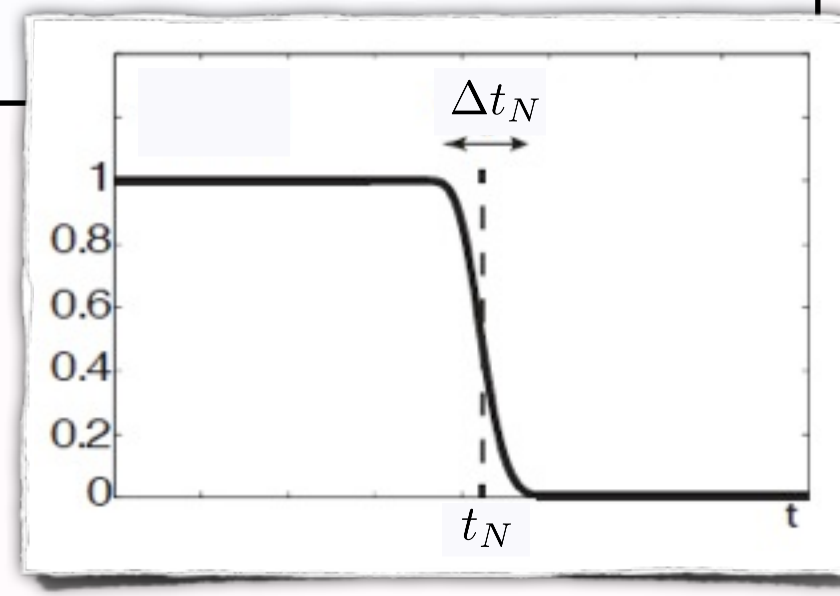
$$\langle 0_N|\mathrm{tr}_{N-1}(e^{t_N(x)\mathcal{L}^{\mathrm{cut}}}(\phi_0))|0_N\rangle = \Phi(x) + \Omega(1/\sqrt{N})$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$ is the cumulative normal distribution function

- Gives a cutoff

$$\lim_{N\to\infty} \langle 0_N|\mathrm{tr}_{N-1}[e^{ct_N(0)\mathcal{L}^{\mathrm{cut}}}(\varphi_0)]|0_N\rangle = 0, \text{ for } c<1,$$

$$\lim_{N\to\infty} \langle 0_N|\mathrm{tr}_{N-1}[e^{ct_N(0)\mathcal{L}^{\mathrm{cut}}}(\varphi_0)]|0_N\rangle = 1, \text{ for } c>1,$$



at time $t_N = N/\gamma$, and even stronger statement, as away from $\Delta t_N = \sqrt{N}$ cutoff window, tails are expressed in terms of Φ

- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Timed dissipation-driven quantum information processing

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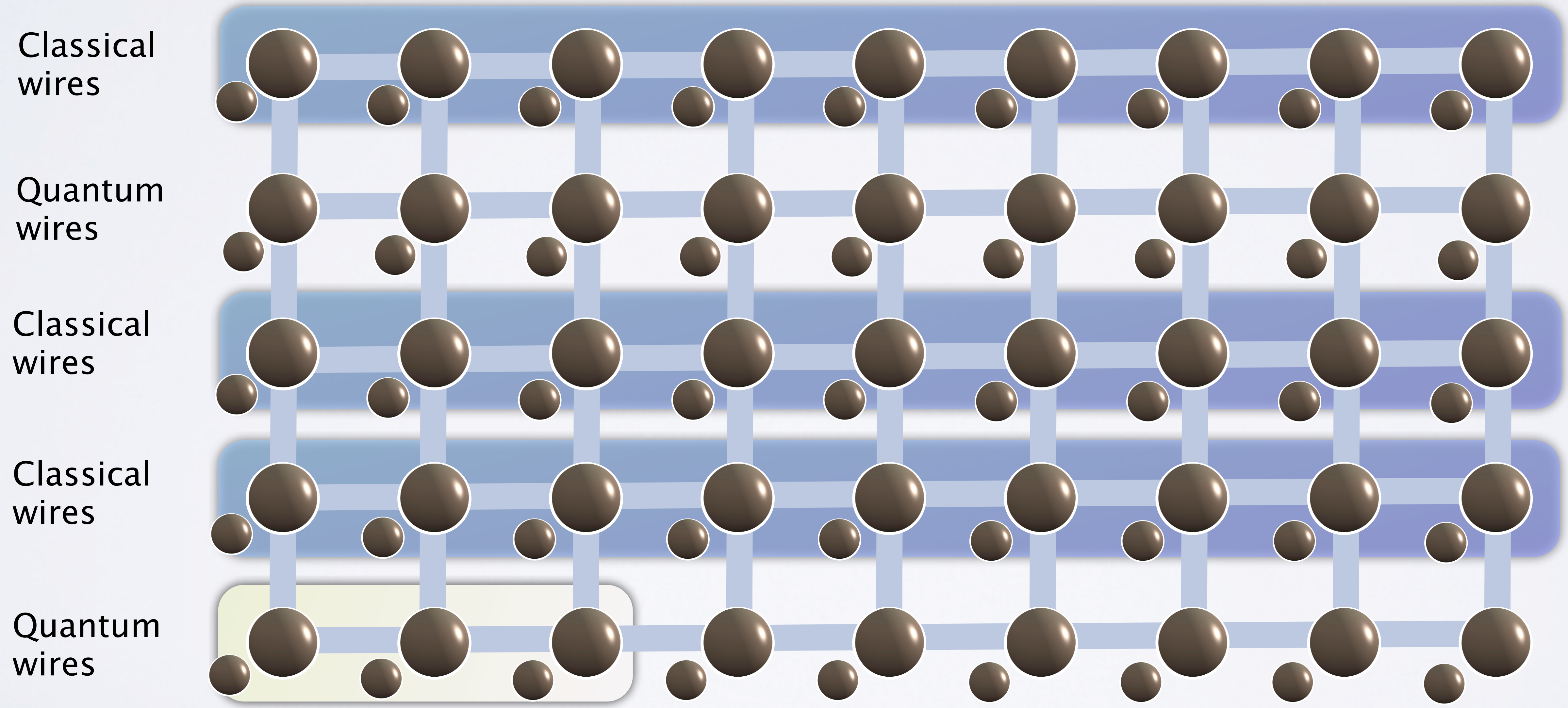
Timed dissipation-driven quantum information processing

- **Measurement-based quantum computing** without measurements
- Classical "wires" propagating classical information



- Raussendorf, Briegel, Phys Rev Lett **86**, 5188 (2003)
- Raussendorf, Briegel, Quant Inf Comp **6**, 433 (2002)
- Hein, Eisert, Briegel, Phys Rev A **69**, 062311 (2004)
- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

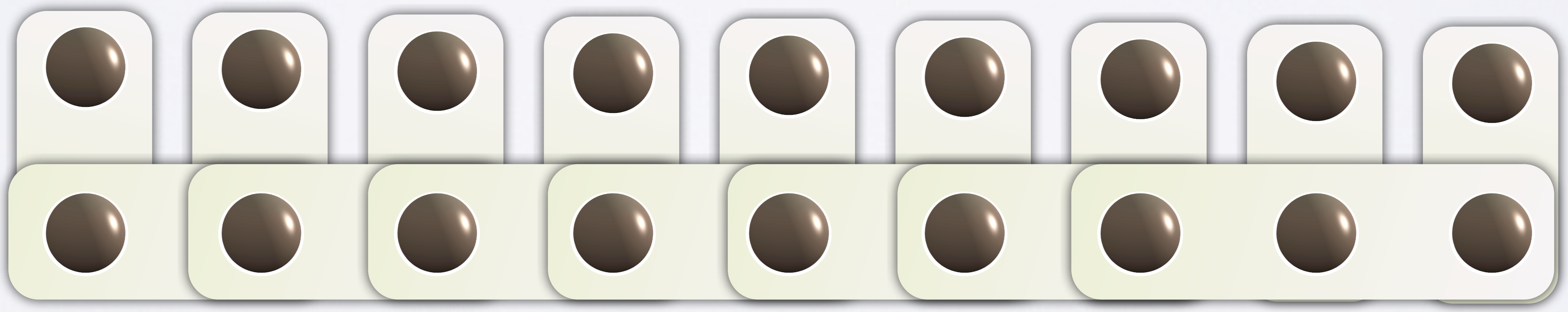
Timed dissipation-driven quantum information processing

- Timer and initialization gadgets can be concatenated, can trigger at integer multiples of cN, c const, at exponentially small accumulated error in N, M

- **Example:**

- Any **graph state** can be prepared in triggered fashion in $O(\log N)$ time

- Combine preparation gadget with Lindblads $L_k = \sqrt{\gamma} Z_k \frac{1 - X_k \prod_{\langle j,k \rangle = 1} Z_j}{2}$



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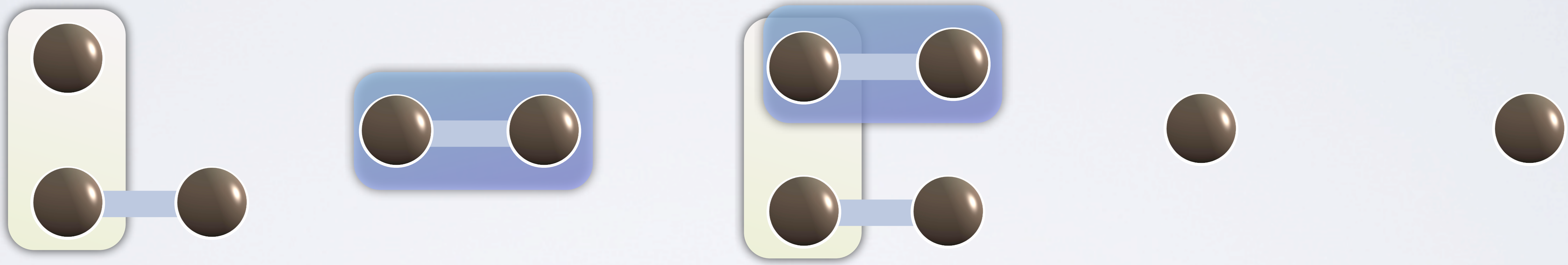
Timed dissipation-driven quantum information processing

- Can emulate any sequence of

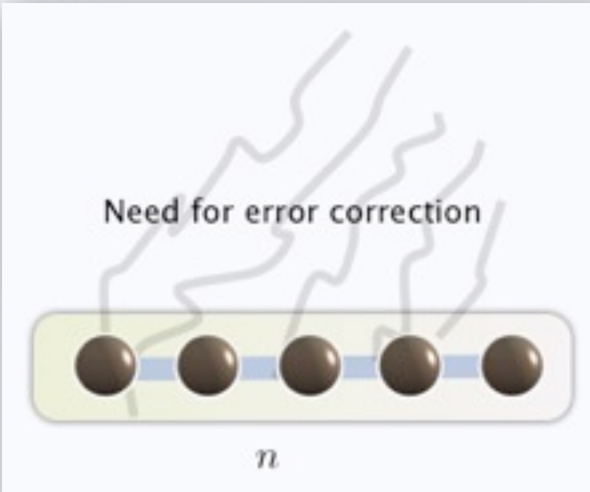


can be done in **dissipative fashion**, with **error bounds** and runtime estimates

- Nice fair computational model, as one cannot hide any of resources

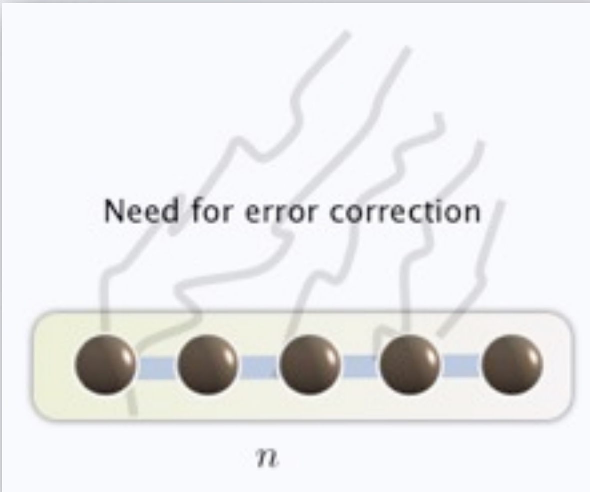
- Nagaj, Wocjan, Phys Rev A **78**, 032311 (2006)
- Vollbrecht, Cirac, Phys Rev A **73**, 012324 (2008)
- Kastoryano, Wolf, Eisert, PRL 110, 110501 (2013)

Summary



Markovian quantum channels and dissipative quantum information processing

Summary



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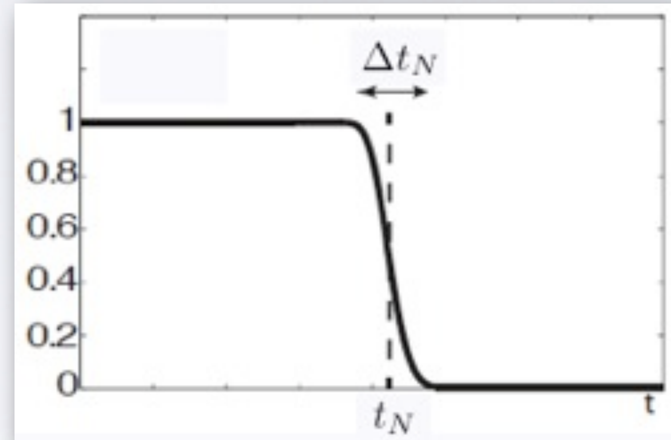

Cutoff phenomenon for Markovian quantum channels

Alexander von Humboldt
Stiftung / Foundation

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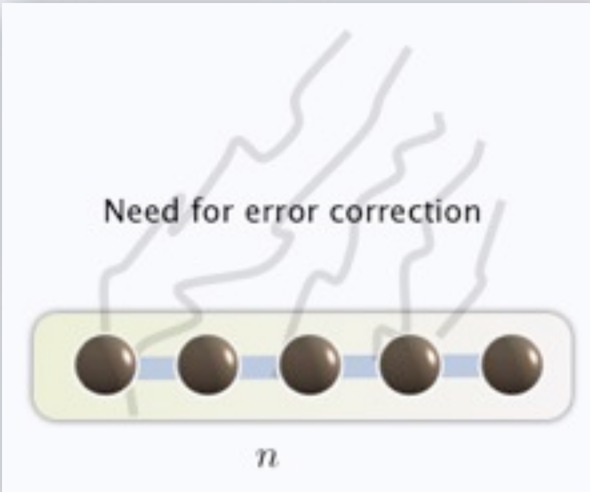

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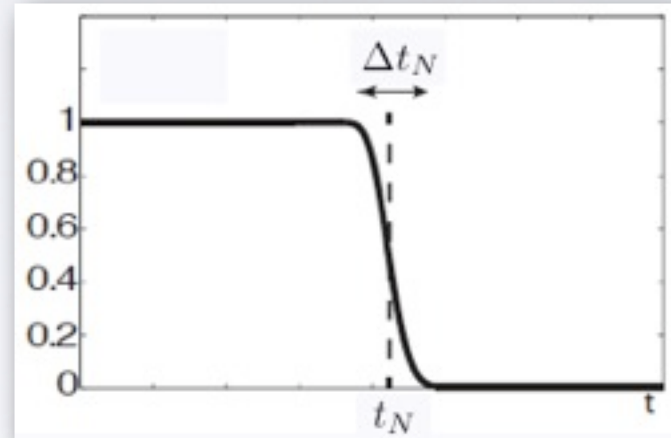

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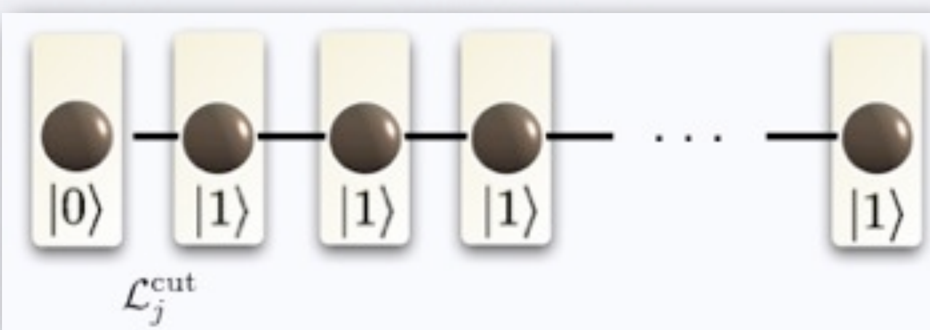

Liouvillian gadgets to construct new schemes, new proof tool

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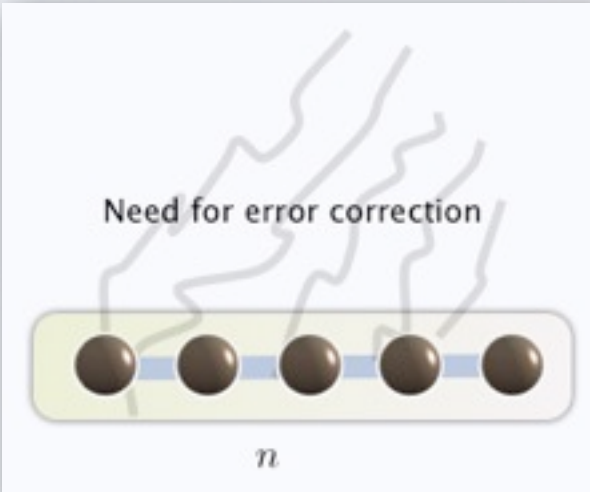

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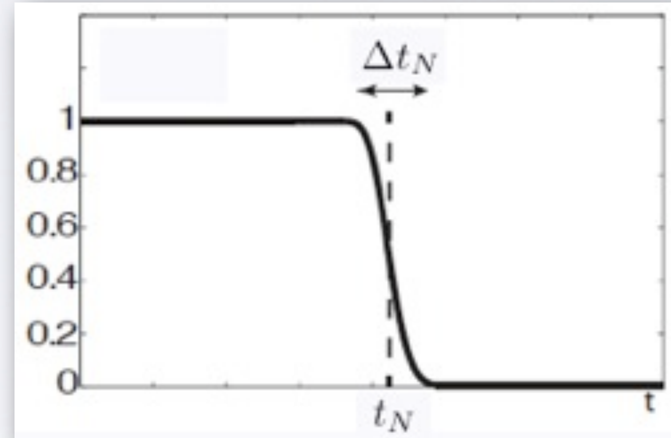

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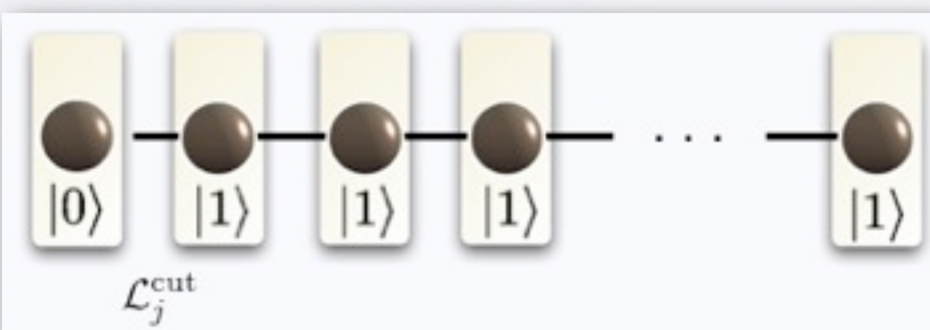

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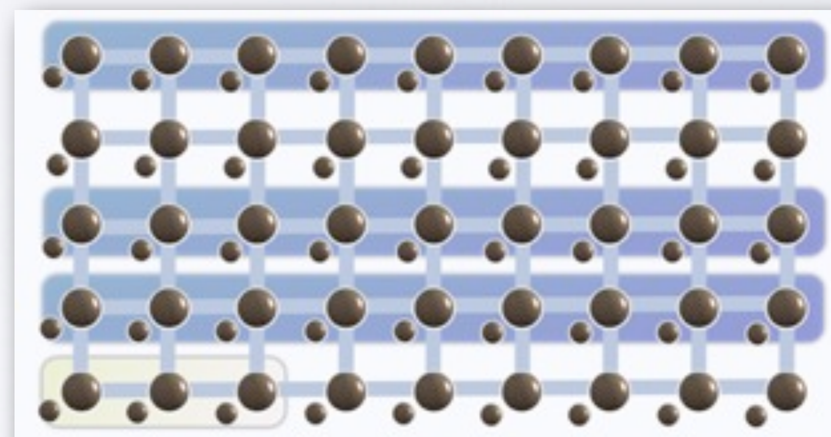

Passive topologically protected quantum memories in 3D?

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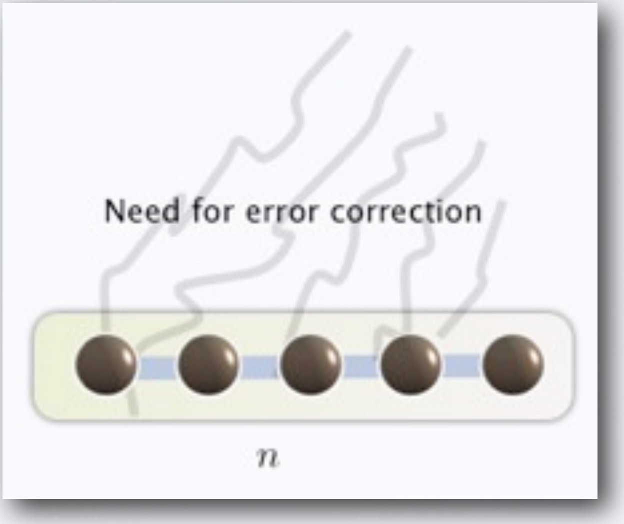
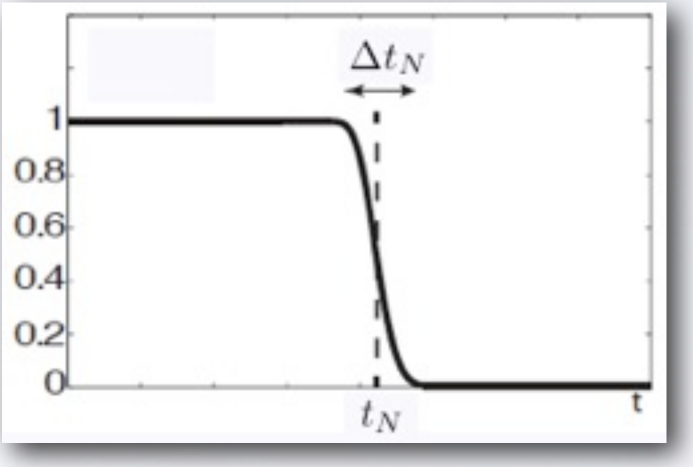
Summary



Markovian quantum channels and dissipative quantum information processing



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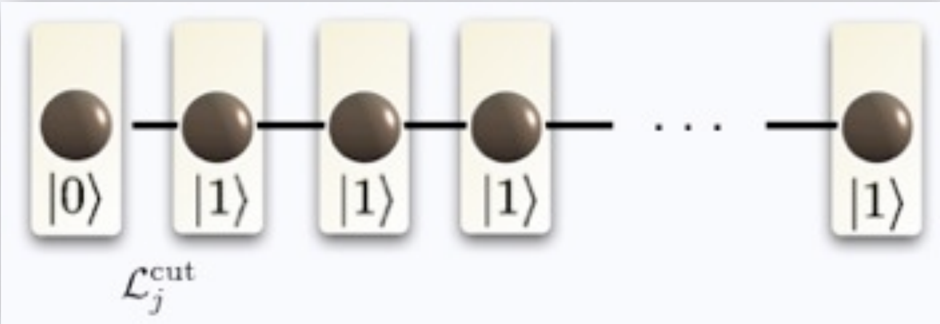

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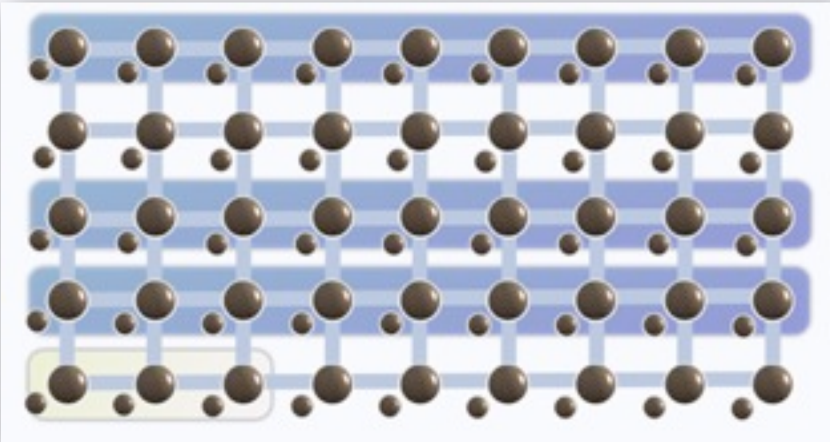

Passive topologically protected quantum memories in 3D?

Thanks for your attention!

Alexander von Humboldt Stiftung/Foundation